

Week 6

Electronic Properties 2

Ohmic and Space Charge Conduction

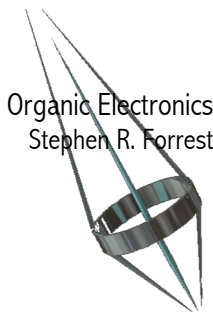
Measuring Mobility

Doping

Recombination

Heterojunctions

Chapter 4.4-4.7



Current and Conductivity

1. Ohm's Law (gives DC mobility).

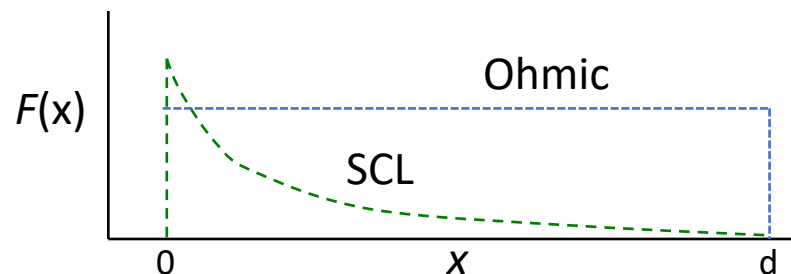
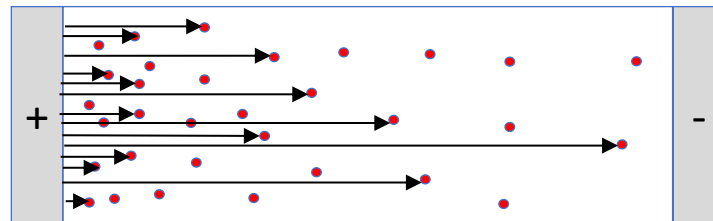
- For a single carrier (holes in this case) in a uniform electric field:

$$j = qp\mu F = qp\mu \frac{V}{d}$$

- Gives the product $p\mu \Rightarrow$ requires independent determination of charge density.
- Ohmic regime* identified by *linear* relationship between j and V .

2. Space charge limited current (gives DC mobility).

- When the injected carrier density $p_{inj} > p_0$ (the background charge density), charge accumulates at electrodes:



Extracting the Diffusion Constant

Shockley-Haynes method (time of flight)

- Bias sample at quasi-equilibrium to avoid injection (Ohmic at $V_a \rightarrow 0$).
- Light pulse generates excitons that separate into charges at $t = 0$
- Measure arrival time (t_D) of the photogenerated current pulse.

$$t_D = \frac{L^2}{\mu V_a}$$

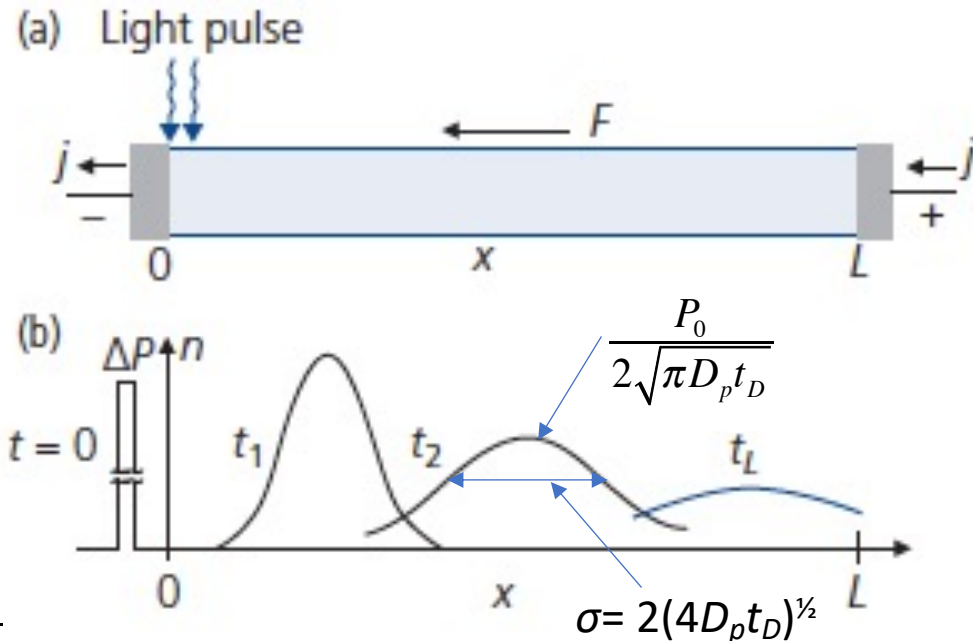
Start with diffusion equation: $\frac{\partial p}{\partial t} = D_p \frac{\partial^2 p}{\partial x^2}$

With solutions: $p(x,t) = \left[\frac{P_0}{2\sqrt{\pi D_p t}} \right] \exp(-x^2 / 4D_p t)$ (A single $\mu \Rightarrow$ Gaussian spreading)

The peak decreases with t_D , and it spreads with half width at $1/e$ from max.: $\sigma = 2(4D_p t_D)^{1/2}$

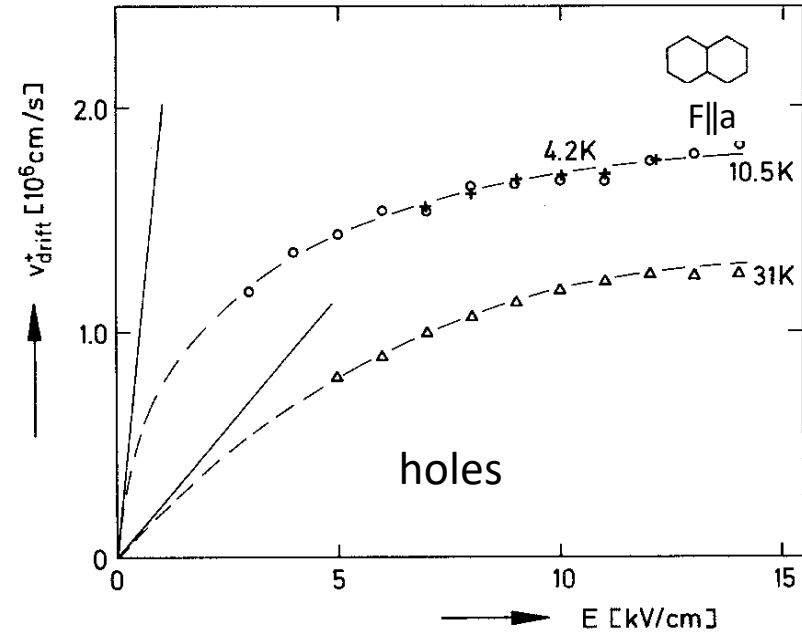
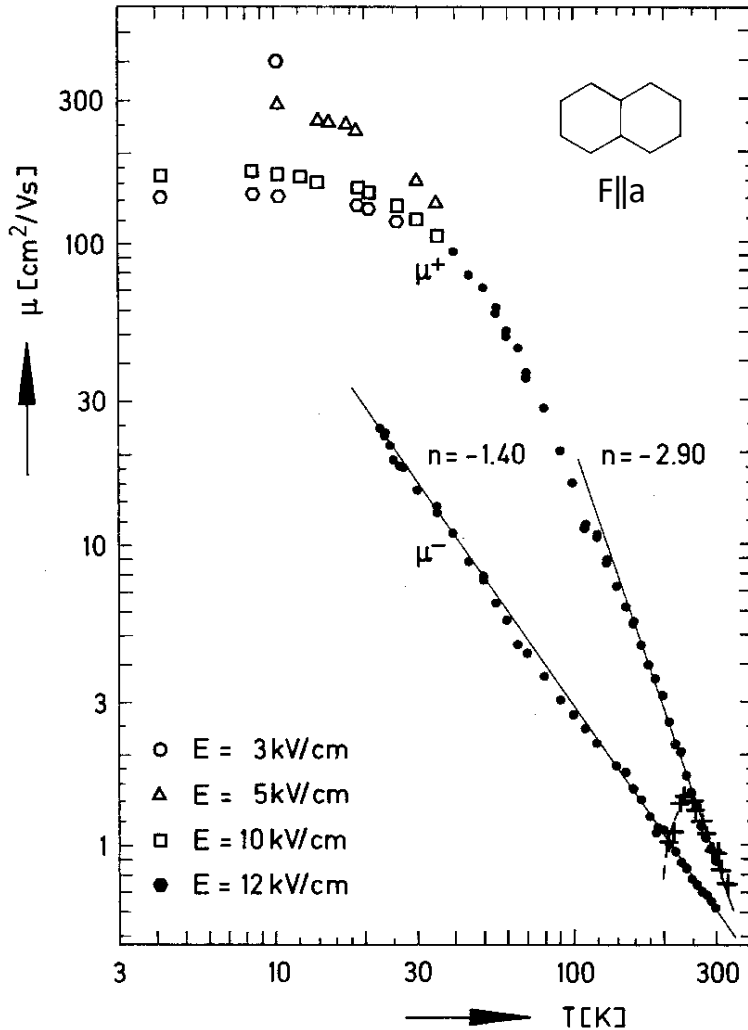
The width of the current pulse gives the diffusion constant of the charge, D .

D_p should be consistent with the Einstein relation $\Rightarrow \mu$



Band Transport

- Ultra-purified naphthalene



- Mobility vs. majority carrier type
 e.g. If the mobility of holes > electrons,
 does NOT imply the material is *p*-type
- The “type” of a material depends on the polarity of the **majority** carrier

Space Charge Limited Current

- In the space charge regime, we make the following assumptions to solve j vs. V :
 - $p_{inj} > p_0$
 - Only one carrier type is present
 - $\mu \neq \mu(F)$ (Field-independent mobility)
 - Free carrier distribution follows Boltzmann statistics
 - Trapped charge occupation defined by Fermi statistics
 - F is large enough for drift (and not diffusion) to dominate
 - Field not so large that field emission is important

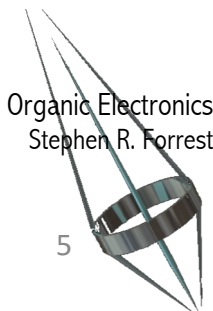
- In 1D, Gauss says:
$$\frac{dF}{dx} = \frac{q(p_{inj}(x) + p_t(x) + p_0)}{\epsilon} \approx \frac{qp_{inj}(x)}{\epsilon} \text{ (trap free case)}$$

- $\epsilon = \epsilon_0 \epsilon_r$

- Current in the absence of trapped charge, $p_t(x)$: $j(x) = q\mu_p p_{inj}(x) F(x)$

- Now:
$$\frac{dF^2(x)}{dx} = 2F(x) \frac{dF(x)}{dx} = \frac{2qp_{inj}(x)F(x)}{\epsilon} = \frac{2j(x)}{\epsilon\mu_p}$$

- Since j is constant across layer $\Rightarrow F^2(x) = \frac{2jx}{\epsilon\mu_p}$ (This is current continuity)



j - V in the SCL regime

$$F^2(x) = \frac{2jx}{\epsilon\mu_p} \Rightarrow F(x) = \sqrt{\frac{2jx}{\epsilon\mu_p}} \quad \left. \vphantom{F(x)} \right\} \text{ Note: } F(x) \sim x^{1/2} \text{ vs. } F(x) = \text{constant for Ohmic}$$

$$\text{Now potential is: } -\frac{dV}{dx} = F(x)$$

Integrating between $0 < V < V_a$ and $0 < x < d$

$$\text{We obtain: } V_a = \frac{2}{3} \sqrt{\frac{2jd^3}{\epsilon\mu_p}}$$

Giving the **Mott-Gurney relationship**:

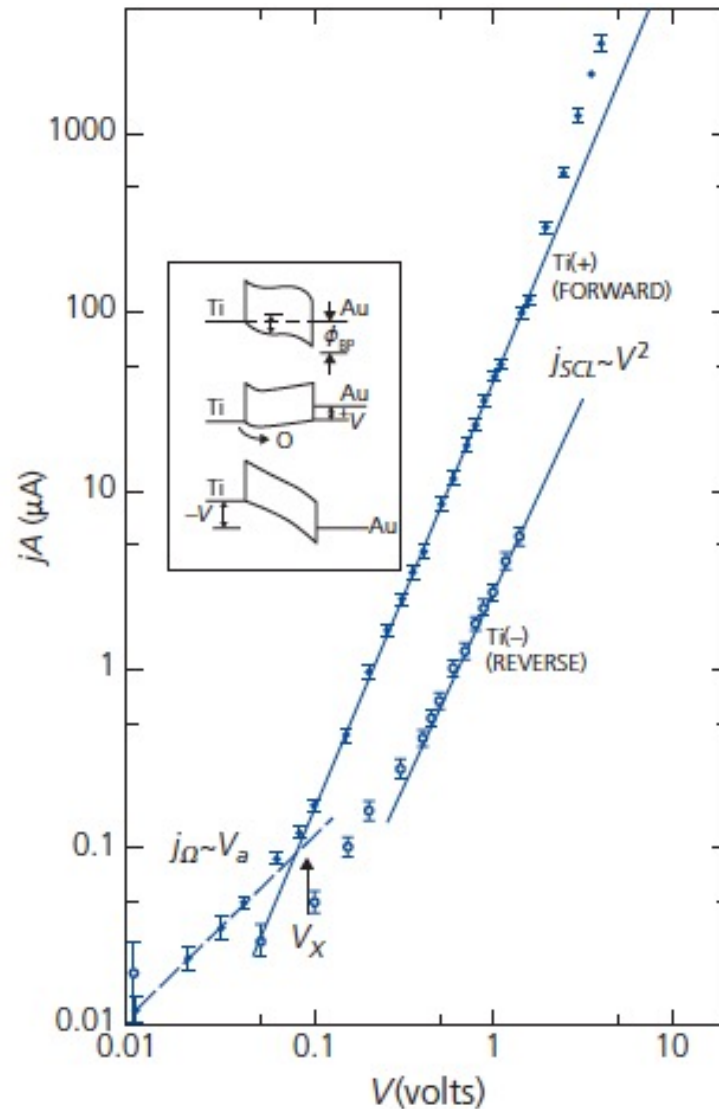
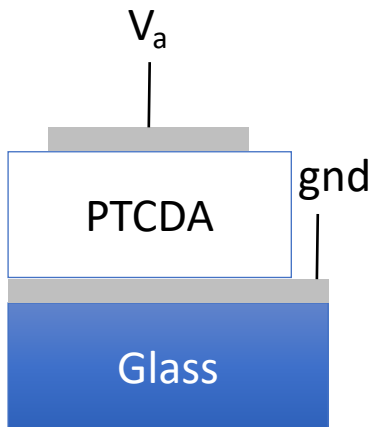
$$j = \frac{9}{8} \mu_p \epsilon \frac{V_a^2}{d^3}$$

Note the absence of p !

\Rightarrow Only need the dielectric constant and the film thickness.

Use the ohmic region of the j - V curve to determine p_0 .

SCL Current in PTCDA



To find background carrier density:

At V_x : $j(\text{ohmic})=j(\text{SCL})$

$$\Rightarrow qn\mu \frac{V_x}{d} = \frac{9}{8} \mu\epsilon \frac{V_x^2}{d^3}$$

$$\Rightarrow n = \frac{9 \epsilon V_x}{8 q d^2}$$



But what happens if things aren't so simple?

- We have assumed no traps. In organics, this is not often the case due to **static disorder**.

- Simplest case: A single discrete, shallow trap where $\frac{p_0}{p_t} = \Theta \ll 1$

- Then you can show: $j = \frac{9}{8} (\Theta \mu_p) \varepsilon \frac{V_a^2}{d^3}$

➤ That is, the mobility is now reduced by Θ

- More often there is an exponential distribution of traps, in which case we have trap-filled limited conduction:

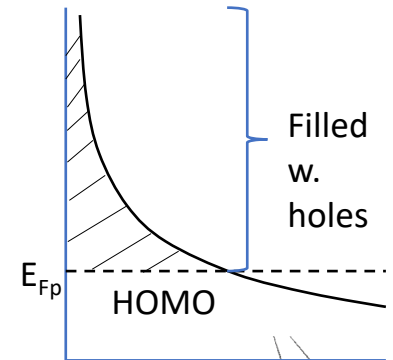
$$j_{TFL} = q\mu N_{HOMO} \left[\frac{\varepsilon m}{q(m+1)N_t} \right]^m \left[\frac{2m+1}{m+1} \right]^{m+1} \frac{V_a^{m+1}}{d^{2m+1}}$$

- $m = T_t/T$ where T_t is the characteristic trap temperature

- Define $p_t = N_t \exp\left(-\left(E_{Fp} - E_{HOMO}\right) / k_B T_t\right)$

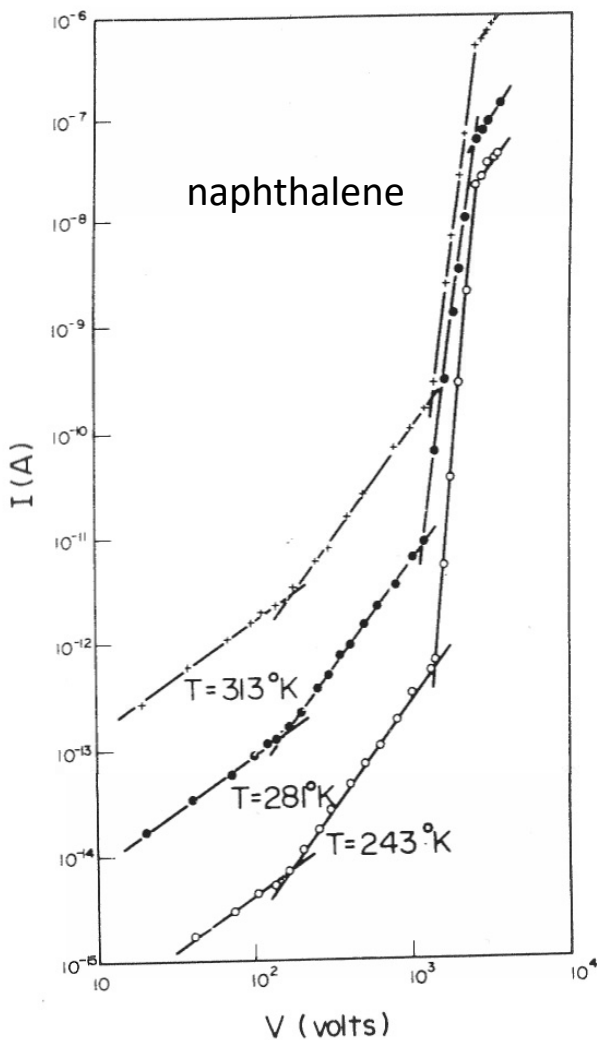
- Leading to: $p = N_{HOMO} \exp\left(-\left(E_{Fp} - E_{HOMO}\right) / k_B T\right) = N_{HOMO} \exp\left(-\left(E_{Fp} - E_{HOMO}\right) / k_B T_t \left[T_t / T\right]\right)$

$$p = N_{HOMO} \left(\frac{p_t}{N_t} \right)^{T_t/T} \Rightarrow p_t = N_t \left(\frac{p}{N_{HOMO}} \right)^{1/m}$$



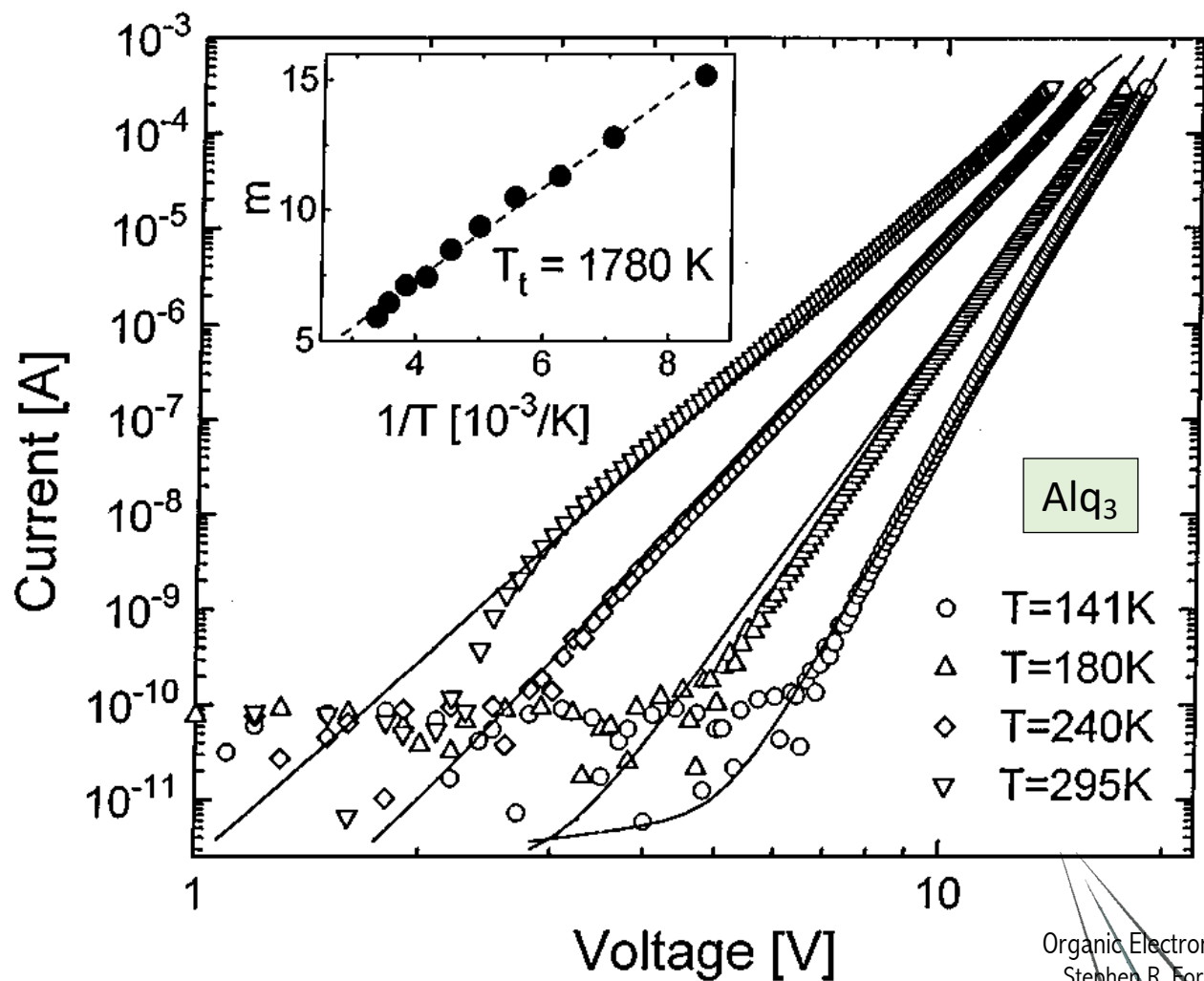
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Examples of TFL-SCL



Multiple ohmic, SCL and TFL regions

M. Campos, Mol. Cryst. Liq. Cryst. **18**, 105 (1972)



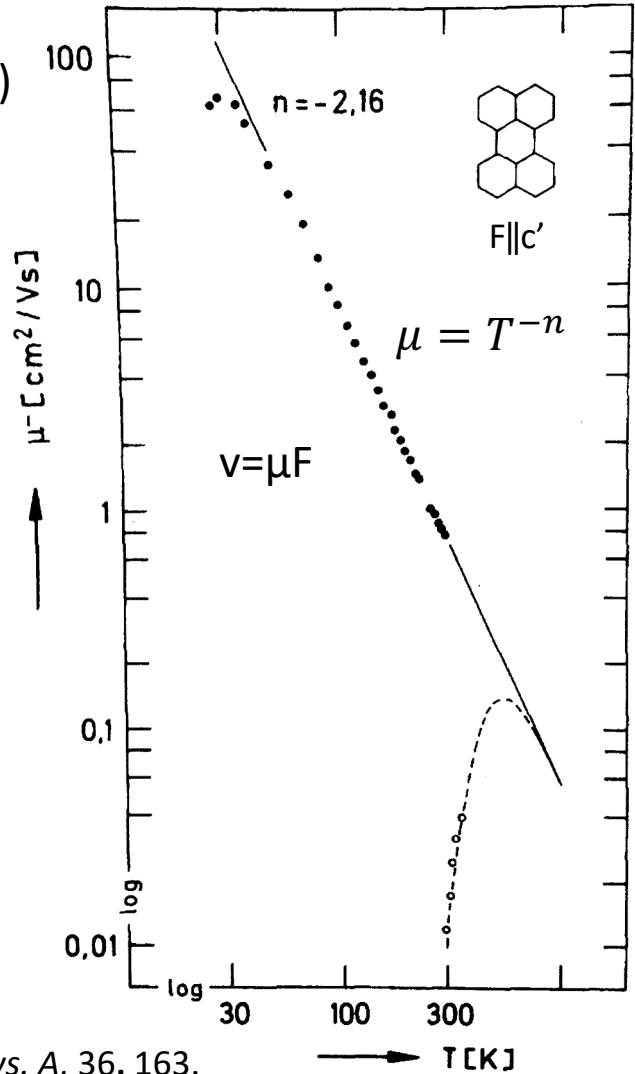
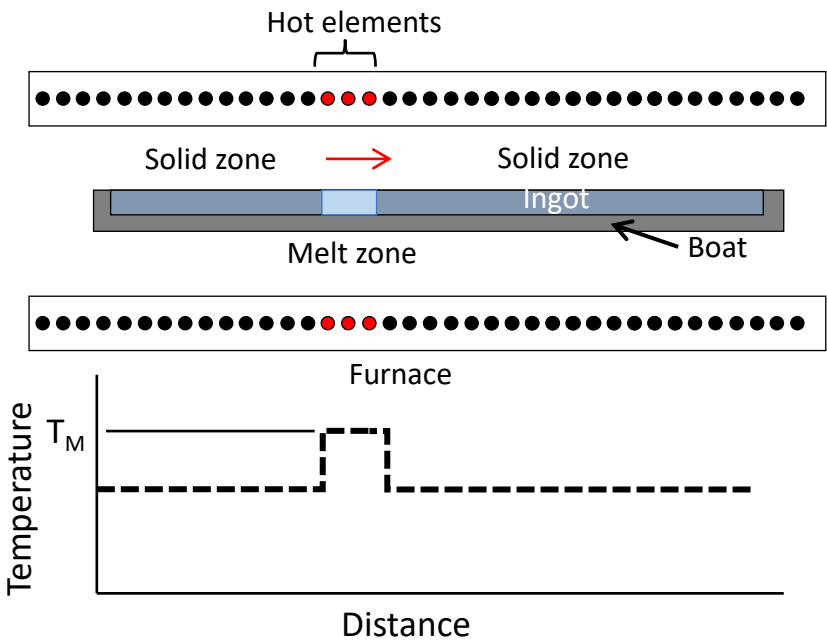
P. E. Burrows, et al., *J. Appl. Phys.*, **79**, 7991 (1996).

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Non-Dispersive Mobility in Ultrapure Organics

$n = 1.5$ band transport
 $n = 2$ acoustic phonon scattering

High purity achieved via zone refining (see Ch. 5)



W. Warta, R. Stehle & N.Karl, 1985. *Appl. Phys. A*, 36, 163.

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Time of Flight Experiment

Ultrapurified Naphthalene Crystals

- Current pulse

$$v > 10^6 \text{ cm/s!}$$

$$\lambda = v\tau$$

$$\tau = \mu m^* / q$$

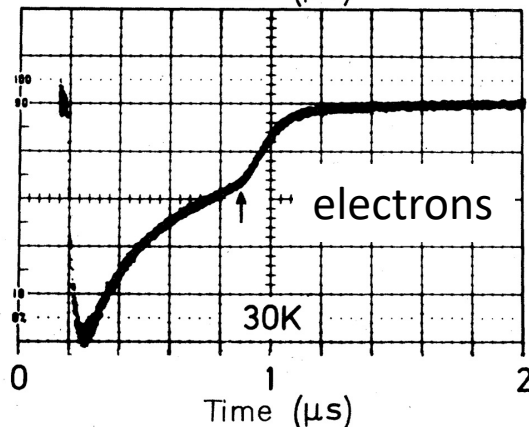
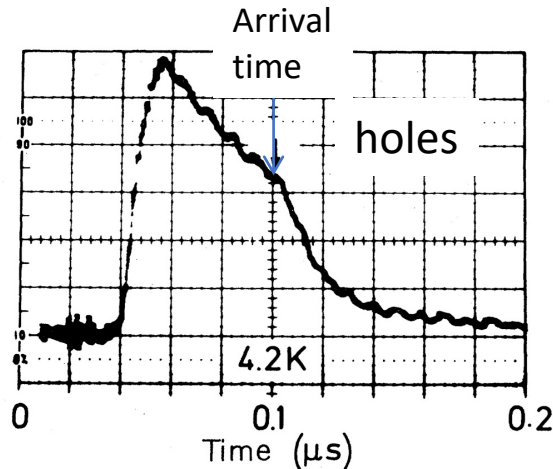
$$v = (3k_B T / m^*)^{1/2}$$

$$\rightarrow \lambda = (\mu / q) (3m^* k_B T)^{1/2}$$

From the data on naphthalene:

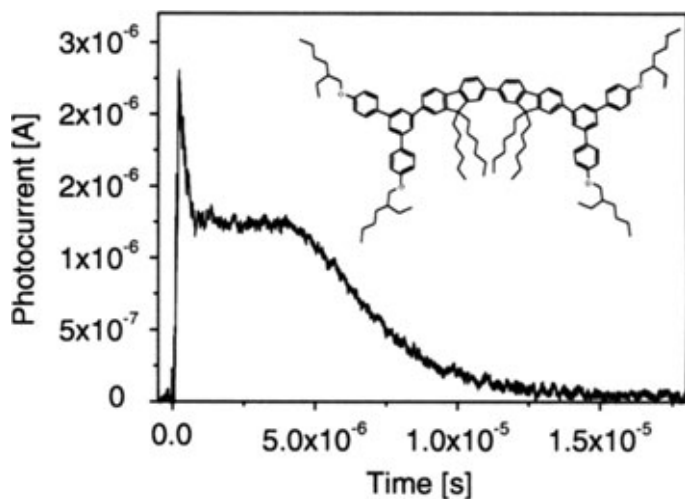
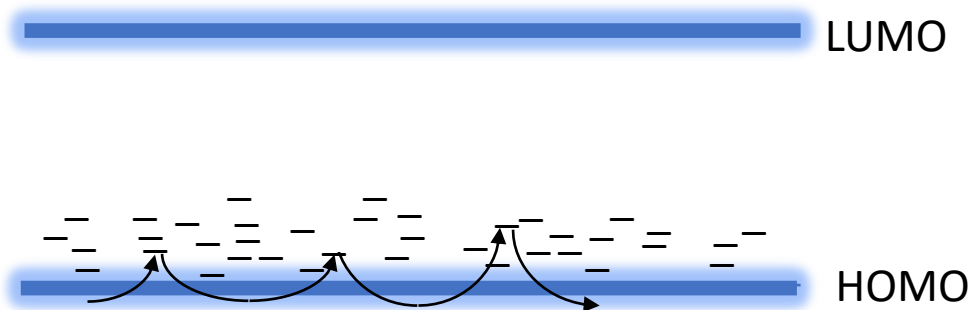
$$m^* \sim 3-15m_0$$

$\lambda \sim 8a$: definitely in the band transport regime



TOF Mobility with Traps

- In the presence of defect states, charges continually trap and de-trap during transit
 - The mobility is not a good number—there are several mobilities, one for each carrier
 - Results in **dispersive transport**

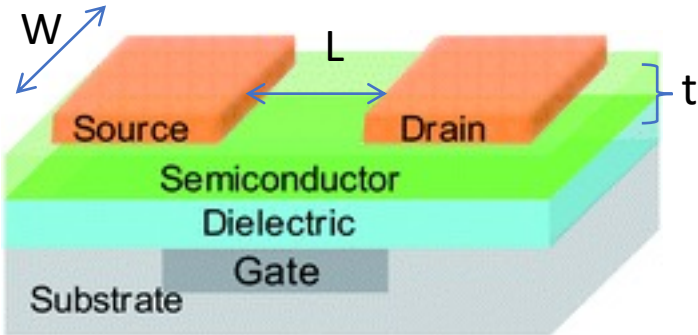


- Initial spike: Charge motion prior to energetic relaxation in the DOS if the RC time constant is short (i.e. reactance small)
- Plateau and broad tail indicate dispersive transport: many different arrival times from trapping/de-trapping during transit.

Measuring Charge Mobility

Transfer characteristics of thin film transistors (OTFTs)

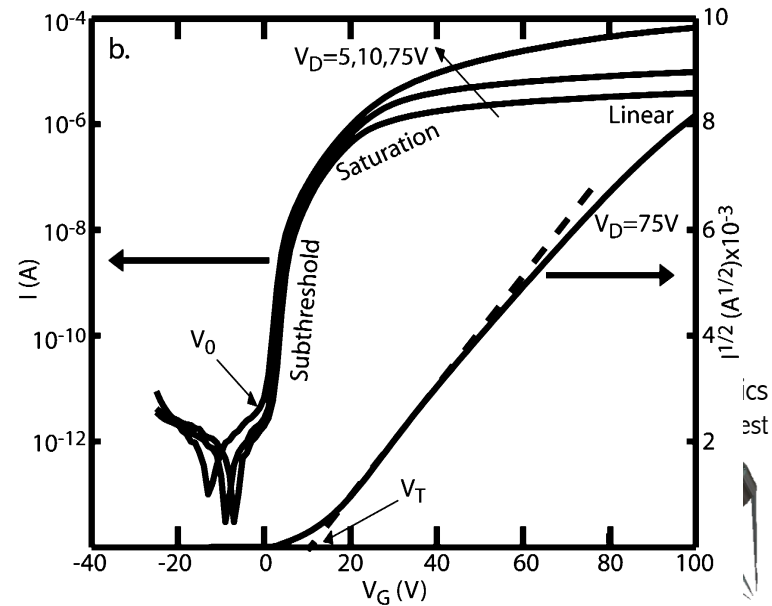
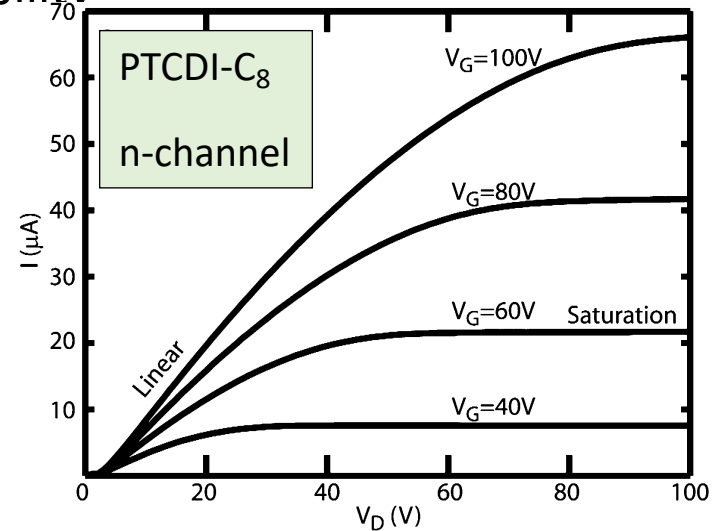
- This measures an interface property, **not** bulk mobility
- Can be strongly influenced by interface trapping
- Can be AC or DC Measurement
 - Almost always used in the less reliable DC mode



Bottom gate, top contact
(BG/TC) source-drain OTFT

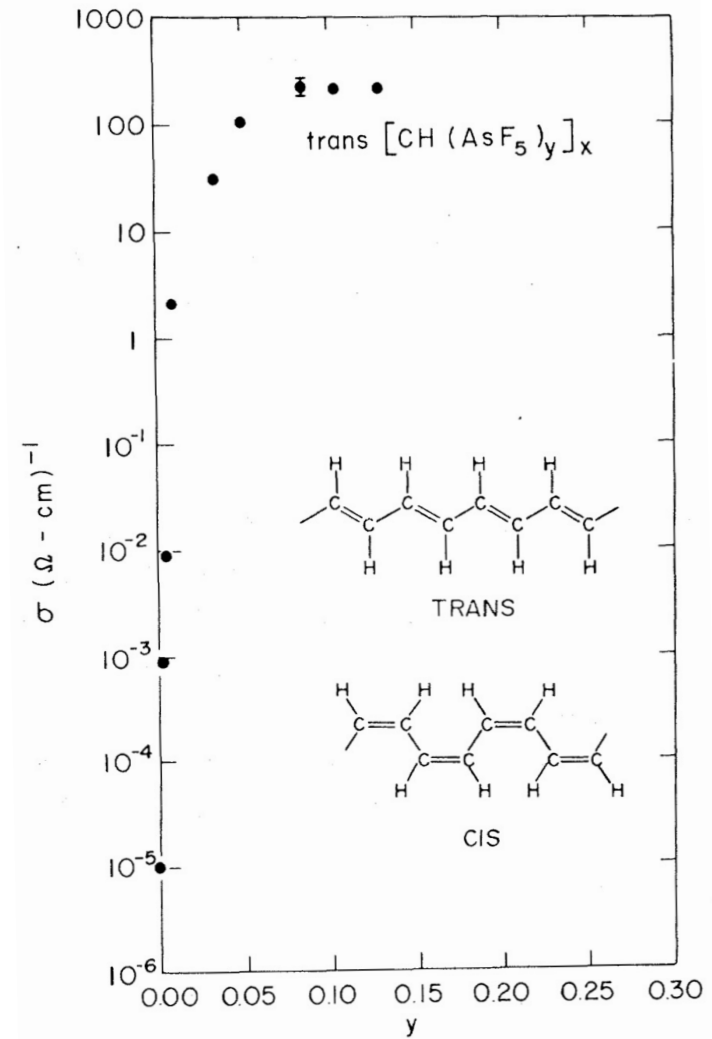
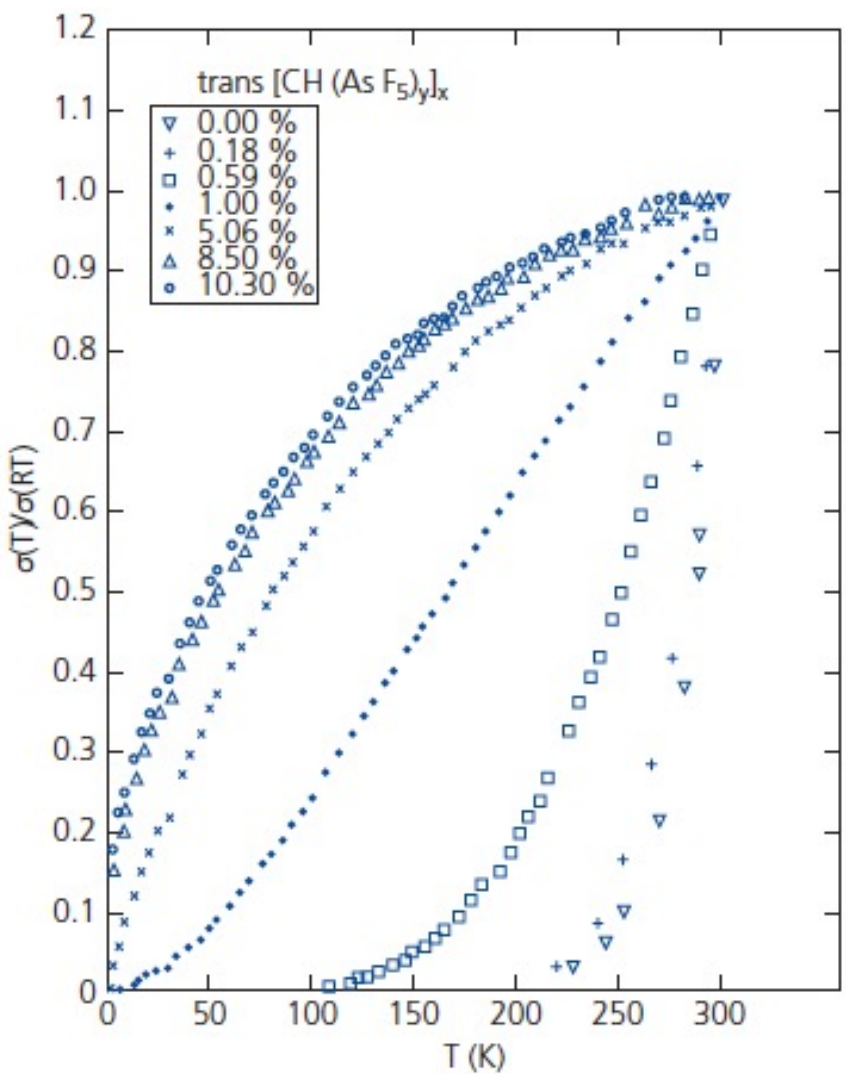
$$I_D = \frac{W}{2L} C_G \mu_{sat} (V_G - V_T)^2 \quad (\text{saturation})$$

Plot of $I_D^{1/2}$ vs. V_G gives both μ_{sat} and V_T



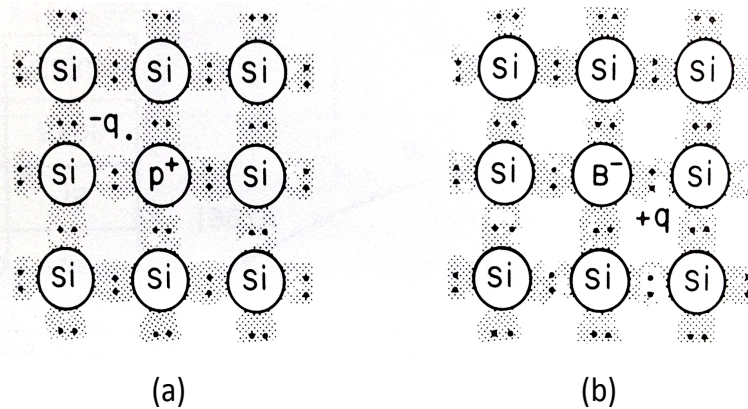
Doping of Organics to Increase Conductivity

Heeger, Shirakawa, MacDiarmid, et al. Phys. Rev. Lett., **39** 1098 (1977)

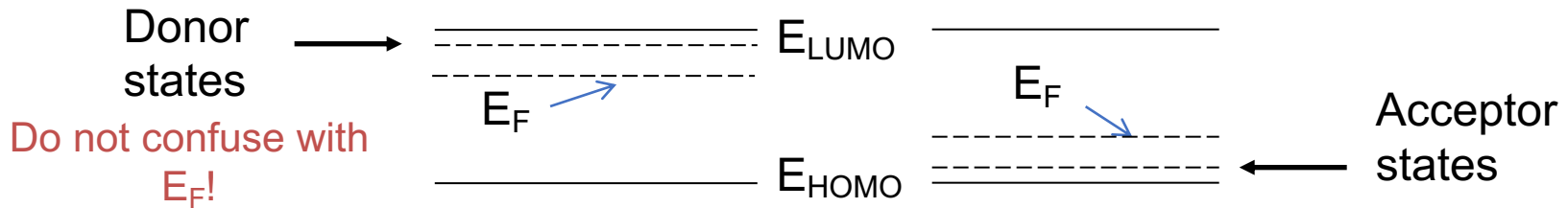


Doping in Organics: Not entirely similar to inorganics

Substitutional doping in inorganics



But no shared bonds in organics!



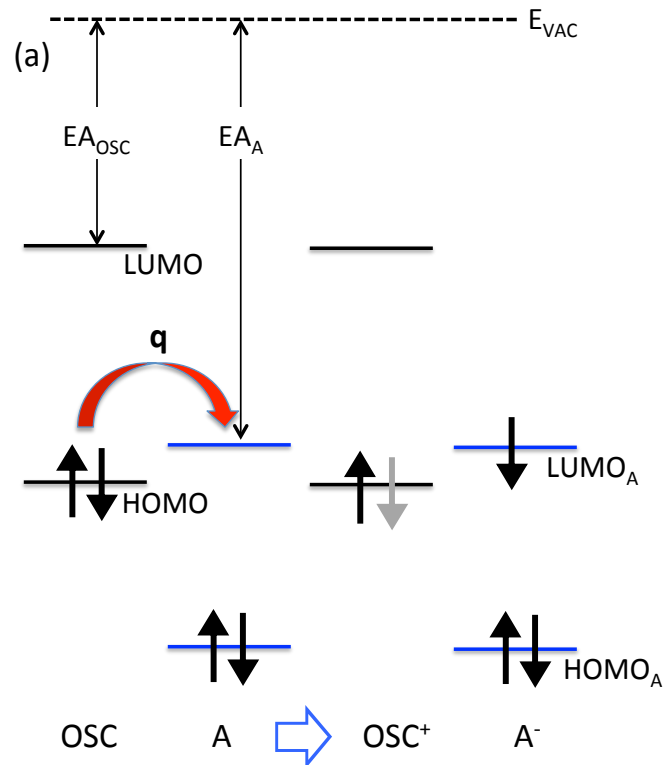
$$n = N_{LUMO} \exp\left(\frac{E_F - E_{LUMO}}{k_B T}\right)$$

$$p = N_{HOMO} \exp\left(\frac{E_{HOMO} - E_F}{k_B T}\right)$$

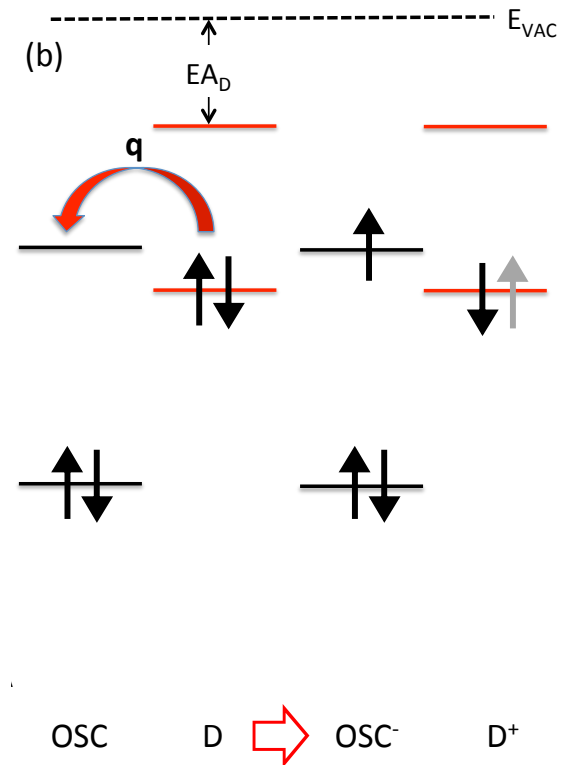
$$\Rightarrow \text{Law of mass action: } n_i^2 = N_{HOMO} N_{LUMO} \exp\left[-\frac{E_G}{k_B T}\right]$$

Doping at the molecular level

Involves charge transfer between dopant and host

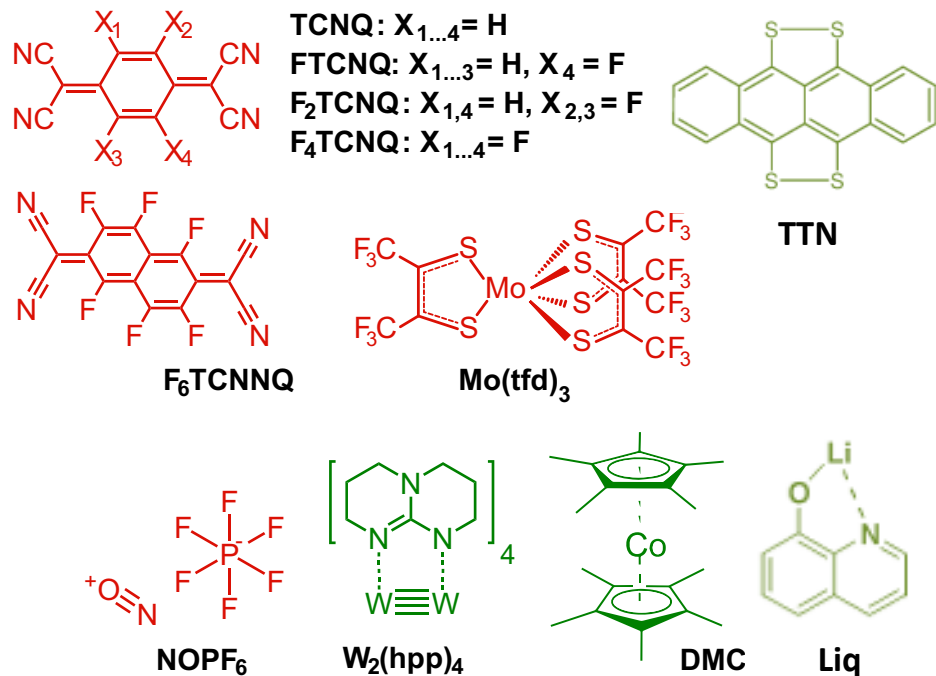


p-type doping



n-type doping

Example *molecular* dopants



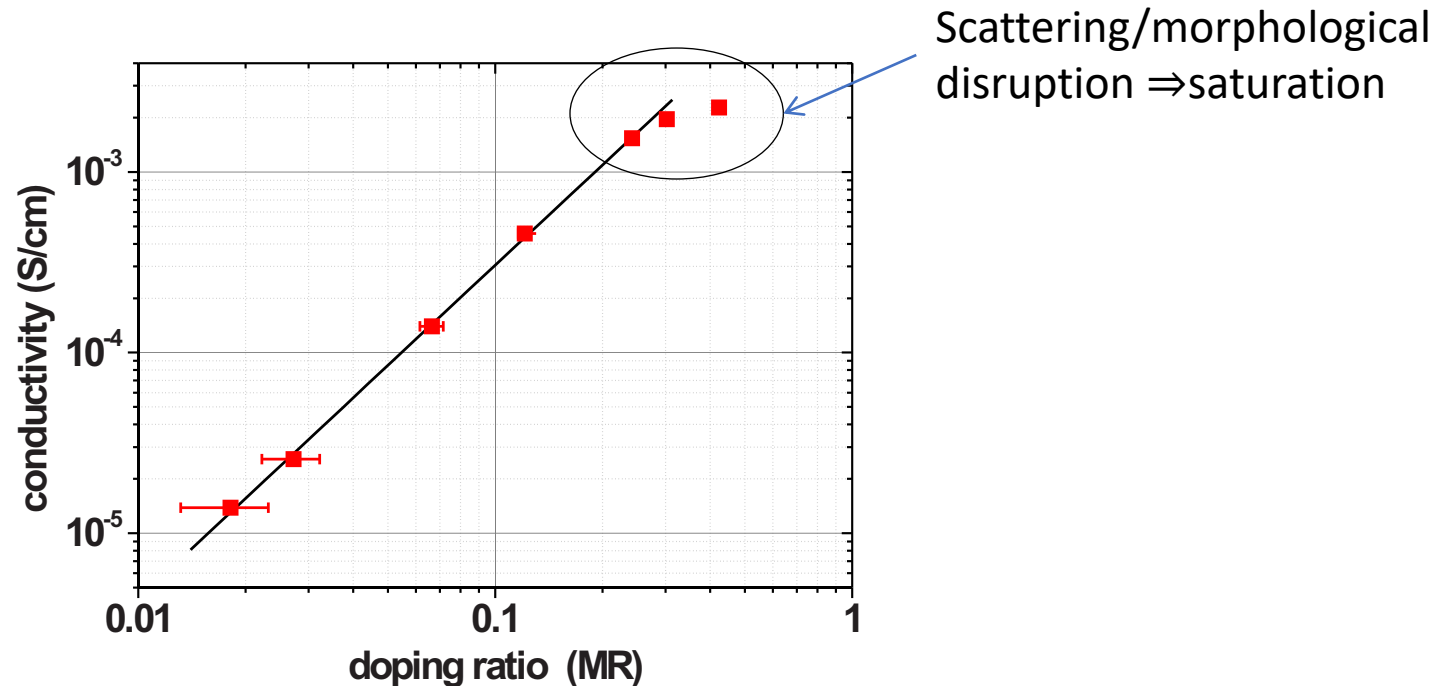
red=acceptors; green=donors

But there are metallic dopants too: Cs, Li, etc.

LiF + Al cathodes
common in OLEDs:



Difficult to get a high conductivity (it takes *a lot* of dopant)



N,N,N',N'-tetrakis(4-methoxyphenyl)-benzidine with **F₄-TCNQ**.

Olthof et al., J. Appl. Phys., 106, 103711 (2009)

Recombination

Charge diffusion equations

$$\frac{\partial n}{\partial t} = \frac{1}{q} \nabla \cdot \mathbf{j}_e - R_e + G_e$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q} \nabla \cdot \mathbf{j}_h - R_h + G_h$$

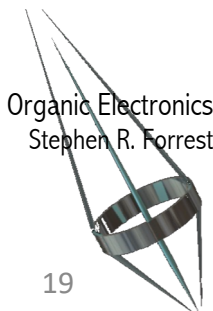
$$\mathbf{j}_e = qD_e \nabla n$$

Using Fick's Law

$$\mathbf{j}_h = -qD_h \nabla p$$

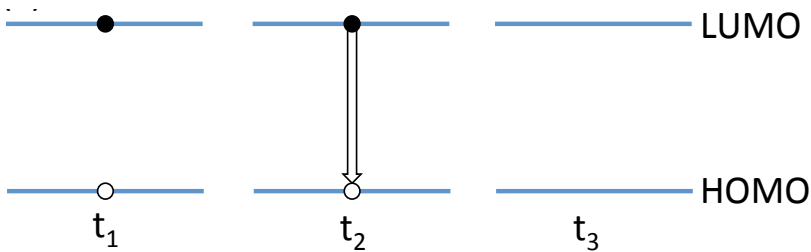
Gives:
$$\frac{\partial n}{\partial t} = D_e \nabla^2 n - R_e + G_e$$

$$\frac{\partial p}{\partial t} = D_h \nabla^2 p - R_h + G_h$$

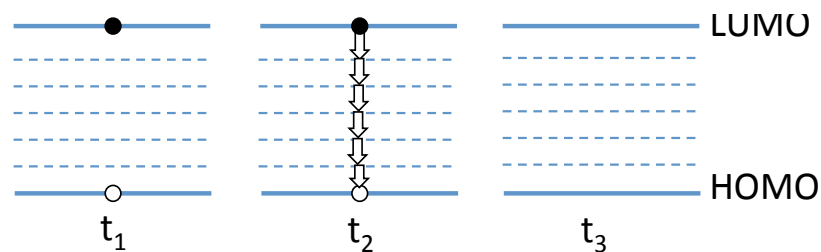


Direct HOMO-LUMO Recombination and via Midgap States

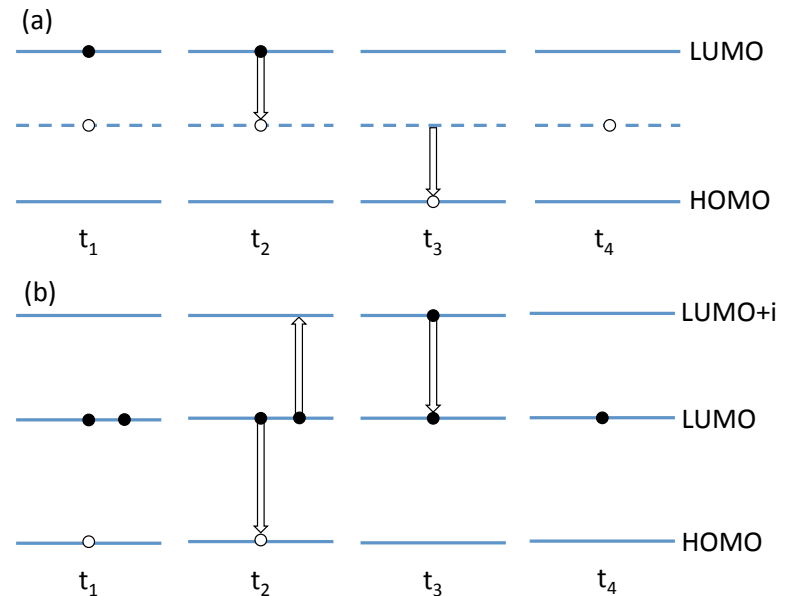
Direct (Band-to Band) Recombination



Multiple Step Recombination



Shockley-Read-Hall Recombination



Auger Recombination

$$R_e^{dir} = \frac{n - n_0}{\tau_e} = \frac{\Delta n}{\tau_e}$$

$$R^{SRH} = \frac{np - n_i^2}{\tau_e(p + p_1) + \tau_h(n + n_1)}$$

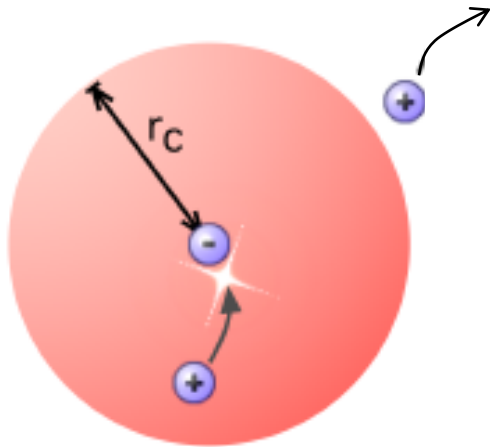
equilibrium restoring force

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Langevin (Bimolecular) Recombination

- When two carriers meet....



Capture radius: When Coulomb = thermal energy

$$r_c = \frac{q^2}{4\pi\epsilon_r\epsilon_0 k_B T}$$

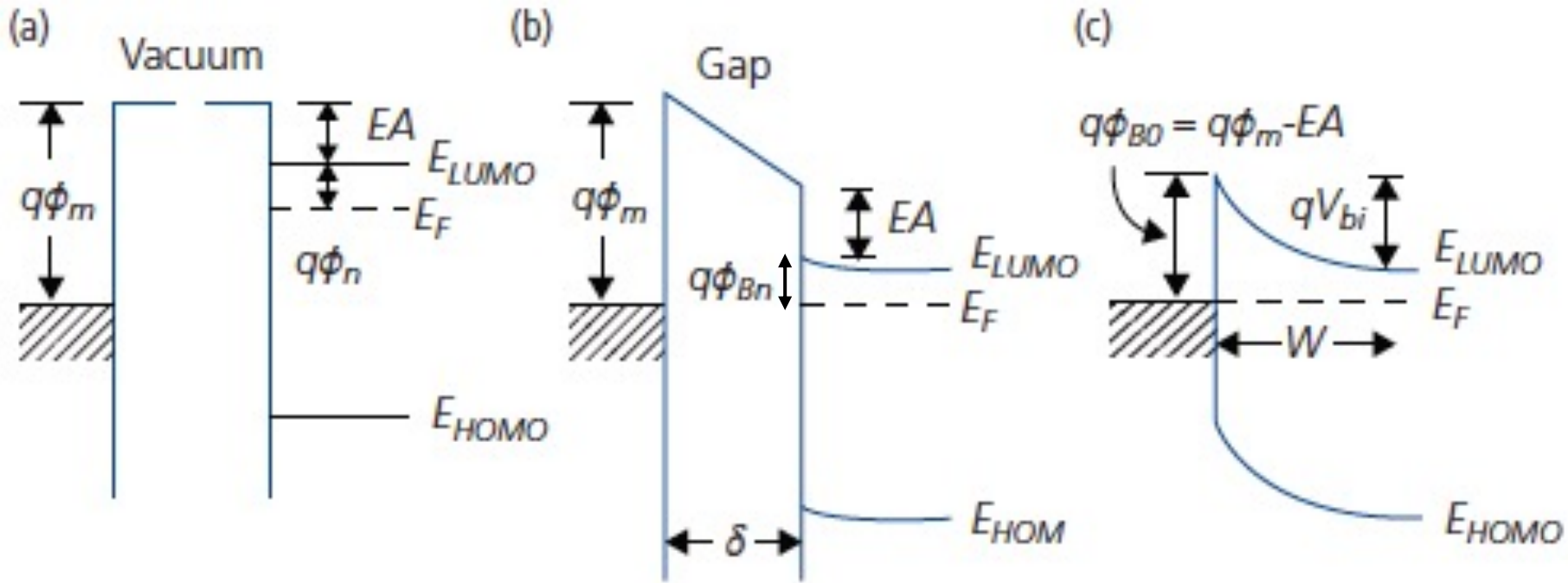
Langevin recombination rate constant:

$$\gamma_L = \frac{q}{\epsilon_r\epsilon_0} (\mu_e + \mu_h) = \frac{q}{\epsilon_r\epsilon_0} \mu_T$$

Yielding the recombination rate (and hence current)

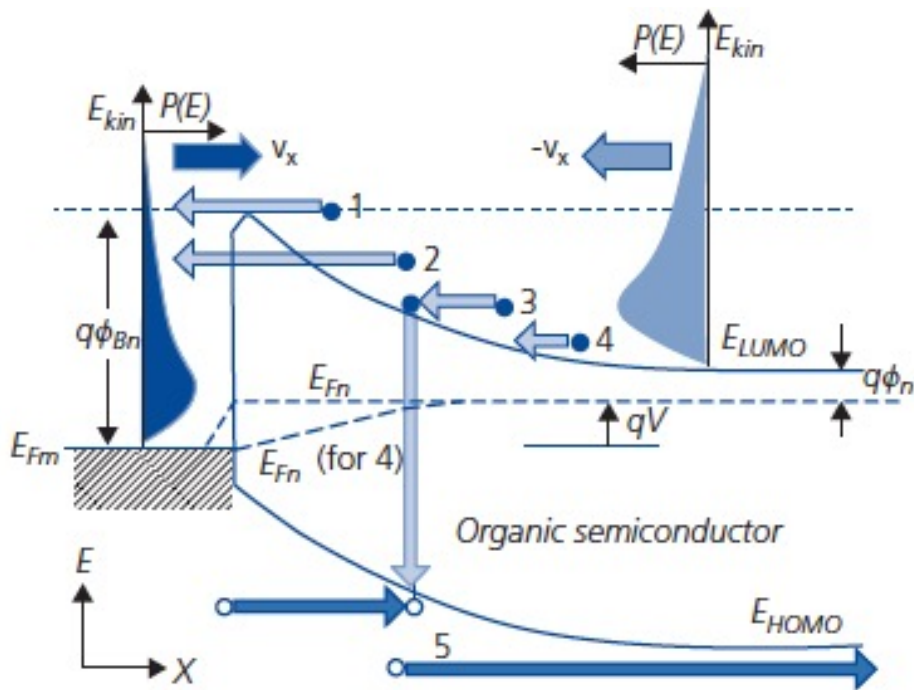
$$R^L = \gamma_L (pn - n_i^2)$$

Schottky barriers



Traps Play a Big Role in Determining Barrier Heights at Metal-Semiconductor Junctions

Current sources across M-O Junctions

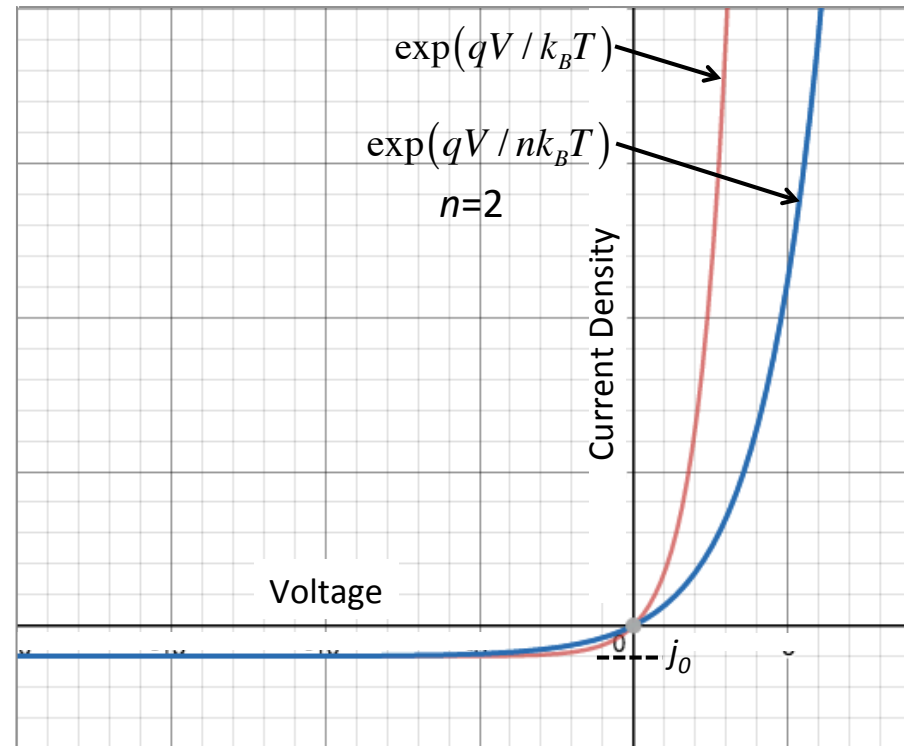


$$j = j_0 \left(\exp(qV / k_B T) - 1 \right)$$

$$j_0 = j_{0TE} = A^* T^2 \exp(-q\phi_{B0} / k_B T)$$

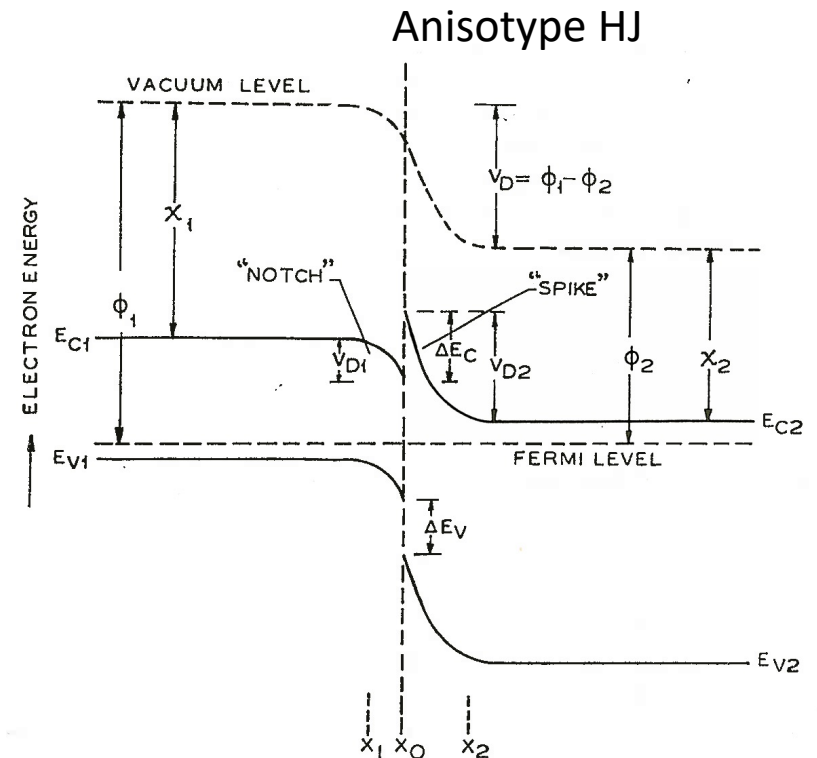
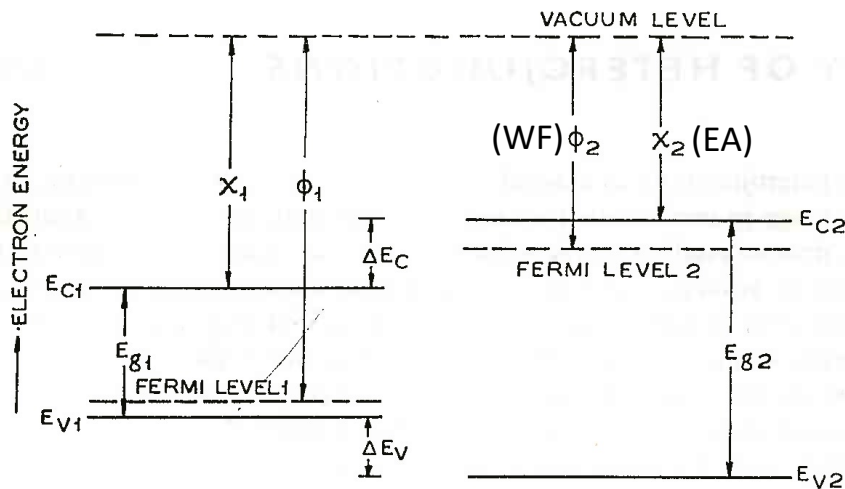
$$A^* = \frac{4\pi q m^* k_B^2}{h^3}$$

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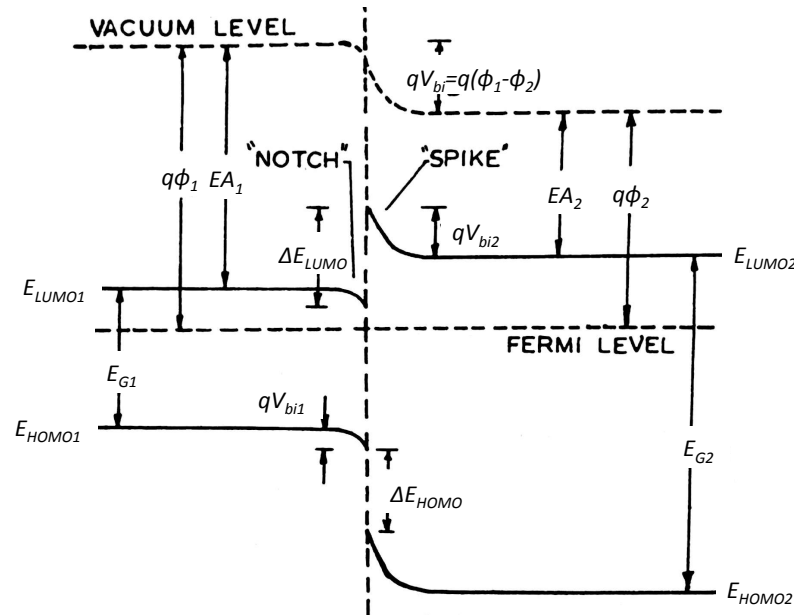
Heterojunctions: Organic-organic contacts

- A heterojunction is a contact between two dissimilar materials (typically semiconductors)
- HJs play a vital role in all photonic devices, and many electronic devices too.
- Some definitions:



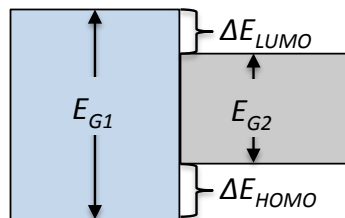
- Anderson's rule: $\Delta E_C = |\chi_1 - \chi_2|$ (doesn't work so well for inorganics due to charge transfer; better for organics)
- $\Delta E_V = \Delta E_g - \Delta E_C$
- Band bending due to free charge: organics tend toward flat bands

Isotype vs. Anisotype HJ

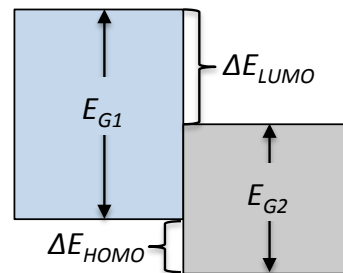


n-N isotype HJ

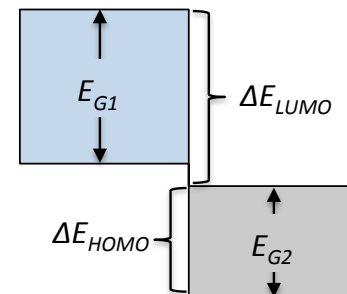
Classification of HJ types



Type I
Nested



Type II
Staggered

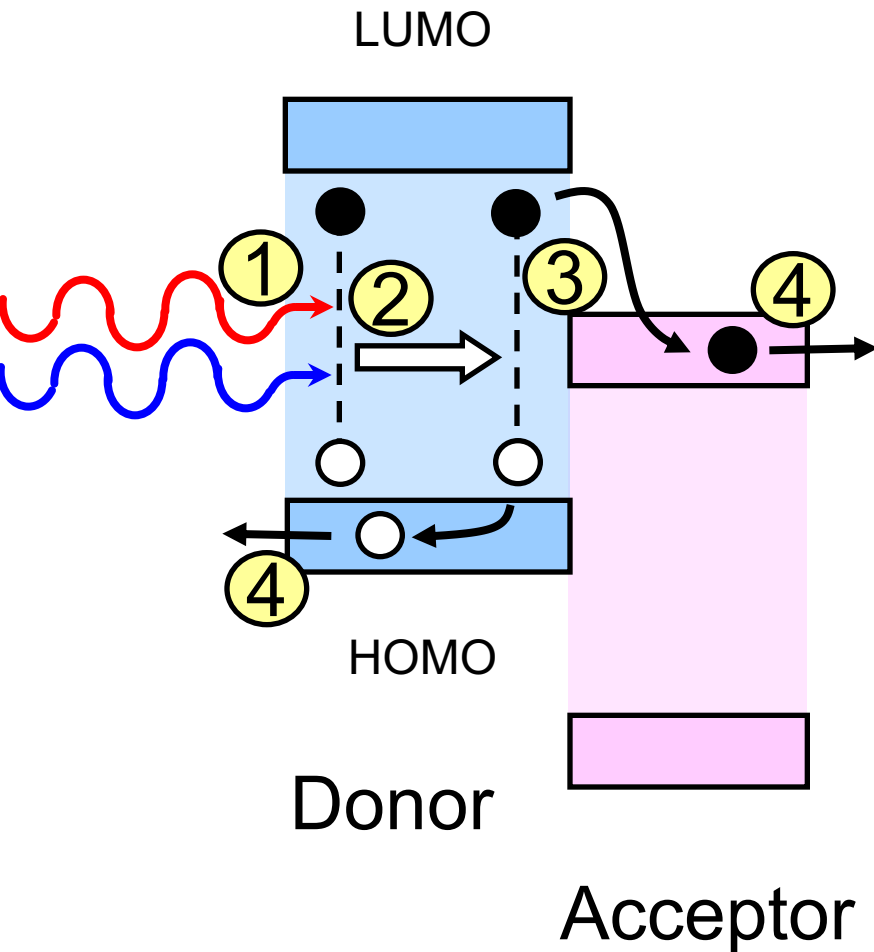


Type III
Broken Gap

Photoinduced Charge-Transfer at a Type II HJ

The Basis of OPV Operation

Processes occurring at a Donor-Acceptor heterojunction



- ① Exciton generation by absorption of light ($1/\alpha$)
- ② Exciton diffusion over $\sim L_D$
- ③ Exciton dissociation by rapid and efficient charge transfer
- ④ Charge extraction by the internal electric field

Typically: $L_D \ll 1/\alpha$

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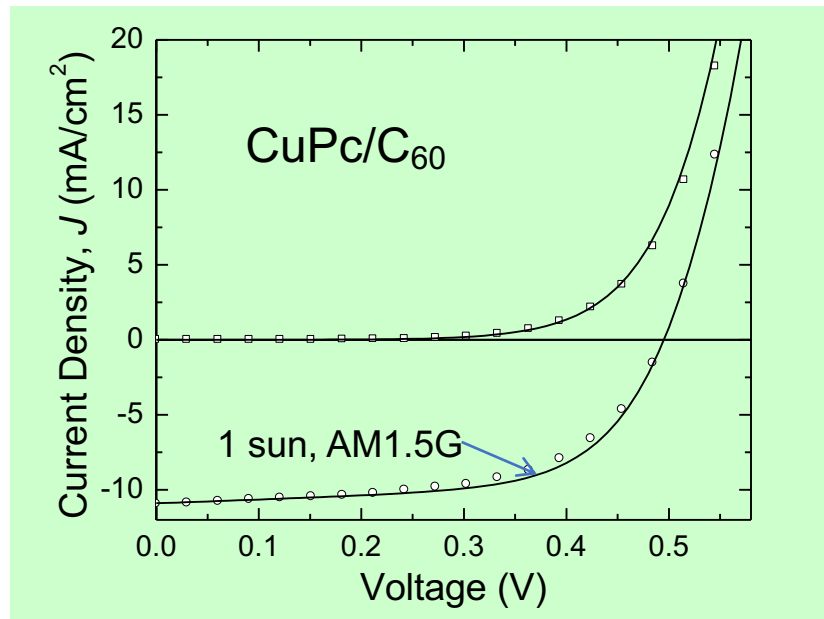
Ideal Diode Equation: Problem Statement

- The Shockley Equation (1949):

$$J = J_o (\exp(qV_a / k_b T) - 1) - J_{ph}$$

has been successfully applied (e.g. Xue and Forrest, 2004) to organic heterojunction cells. But the physics is wrong!

- Why does it “work”?
- Is there a more appropriate relationship for organic (i.e. *excitonic*) HJs?



Excitonic Heterojunctions:

Controlled by **energy transport**, *not* charge transport

1. Excitons diffuse with current J_X to HJ
2. Separate into polaron pair across HJ
3. PP can either dissociate into carriers
4. Or recombine to ground state

ζ =PP density

k_{PPr} =PP recombination rate

k_{PPd} =PP dissociation rate

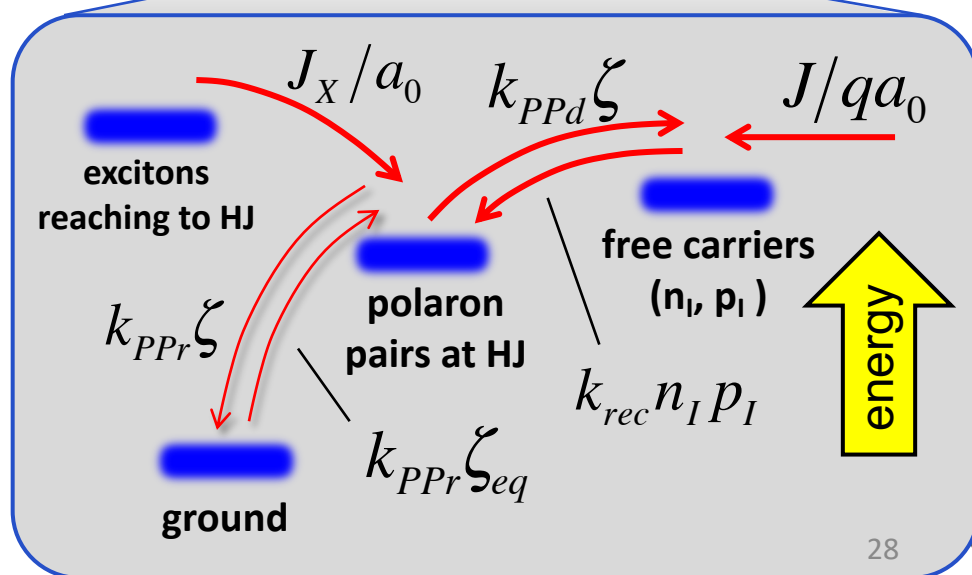
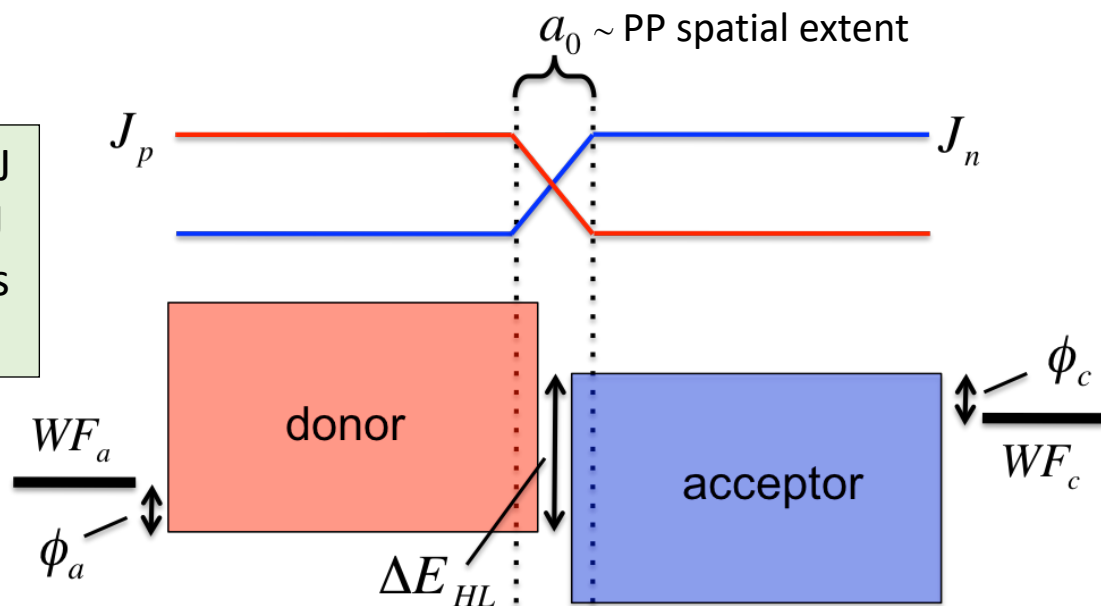
k_{rec} =charge recombination rate

J =electron current

WF =work function

n_I, p_I =charge at interface

A polaron pair at the interface is equivalent to a charge transfer (CT) state



Derivation of the Ideal Diode Eq.

“Just because you have an ideal diode equation does not mean you have an ideal diode”

- The rate equations in steady state:

- Excitons: $\frac{J_X}{a_0} - k_{PPr}(\zeta - \zeta_{eq}) - k_{PPd}\zeta + k_{rec}n_{IP} = 0,$

- Polarons: $k_{PPd}\zeta - k_{rec}n_{IP} + \frac{J}{qa_0} = 0,$

Rate Equations + Fermi Stats:

$$J_0 \left\{ \exp(qV_a / k_B T) - \frac{k_{PPd}}{k_{PPd,eq}} \right\} - J_{ph}$$

$$J = qa_0 k_{rec} N_{HOMO} N_{LUMO} (1 - \eta_{PPd}) \exp(-\Delta E_{HL} / k_b T) \left\{ \exp(qV_a / k_b T) - \frac{k_{PPd}}{k_{PPd,eq}} \right\} - q\eta_{PPd} J_X$$

electron & hole
DOS

PP dissociation
efficiency

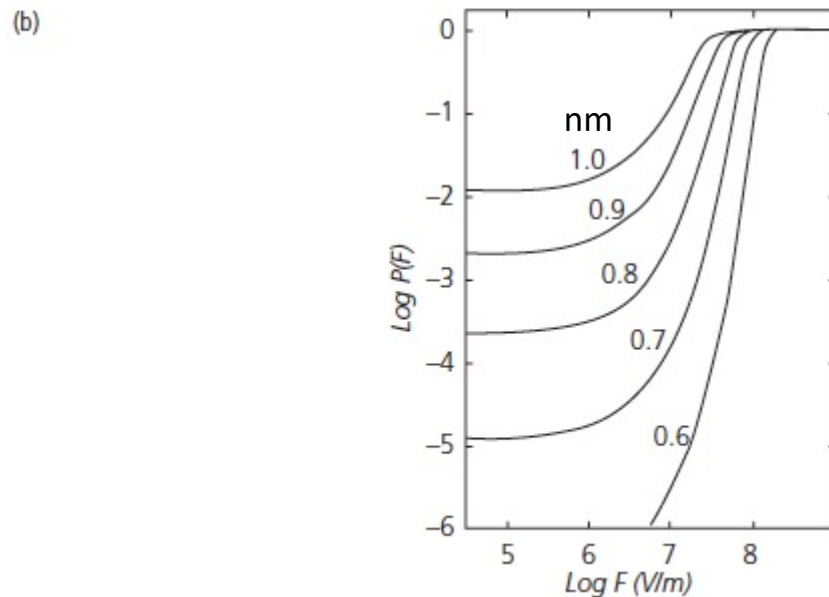
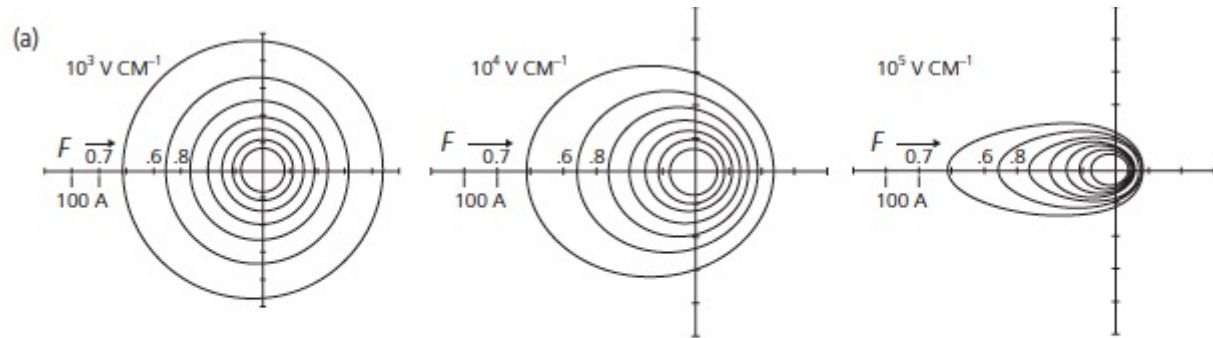
$$\left[\eta_{PPd} = \frac{k_{PPd}}{k_{PPr} + k_{PPd}} \right]$$

equilibrium dissociation
rate

Organic Electronics
Stephen R. Forrest

Onsager-Braun Exciton Polarization

- Why there is a voltage dependence to k_{ppd} that gives j - V slope under reverse bias



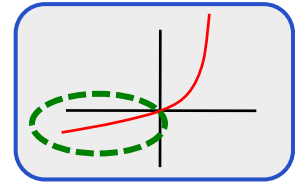
Probability for exciton ionization

Consequences of the diode equation

$$J_0 \left\{ \exp(qV_a / k_B T) - \frac{k_{PPd}}{k_{PPd,eq}} \right\} - J_{ph}$$

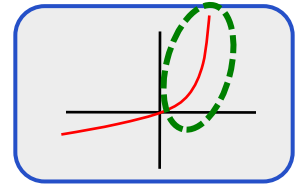
Reverse Bias:

- strong dissociation: $k_{PPd} > k_{PPd,eq}$ → saturation current increases →



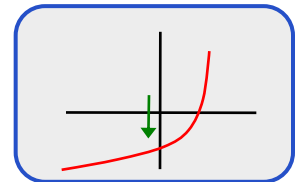
Forward Bias:

- weak dissociation: $k_{PPd} < k_{PPd,eq}$ → exponential diode current →



Illumination:

- photogenerated PPs: $J_X, \eta_{PPd} > 0$ → photocurrent addition →



N. C. Giebink, et al. Phys. Rev. B, **82**, 155305 & 155306 (2010).

Including traps

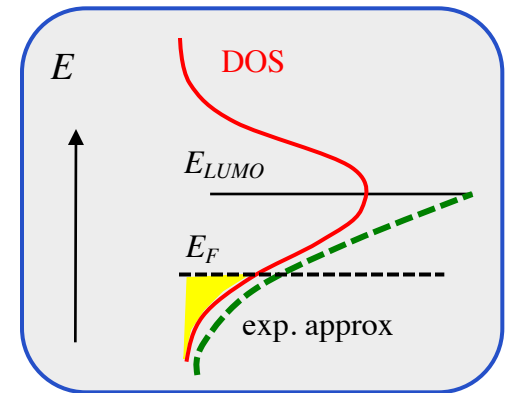
Disordered materials:

- broad density of states (DOS) \Rightarrow continuous trap distribution:

Trap Distribution Function

$$n_t \approx H_A \exp\left(\frac{E_{Fn} - E_{LUMO}}{k_b T_{t,A}}\right) \approx H_A \left(\frac{n}{N_{LUMO}}\right)^{1/l_A}$$

$$\text{where } l_A = T_{t,A}/T \Rightarrow n_A = \frac{l_A}{\delta_D(l_A - 1) + 1}$$

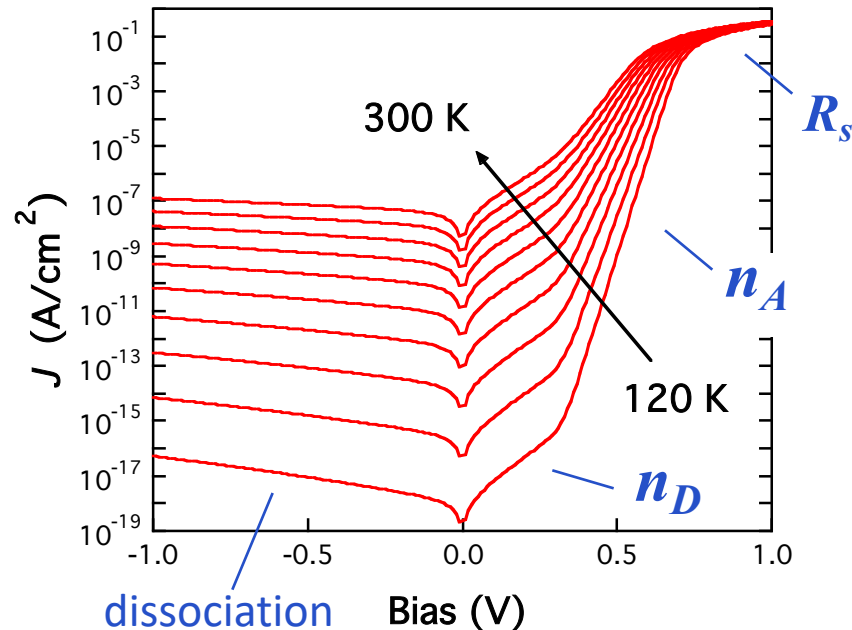


- Ideality factors: n_D , n_A depend on *shape* of trap DOS
 - e.g. $n=2$ for uniform distribution between HOMO and LUMO

Dark Current With Traps

- General form including series resistance:

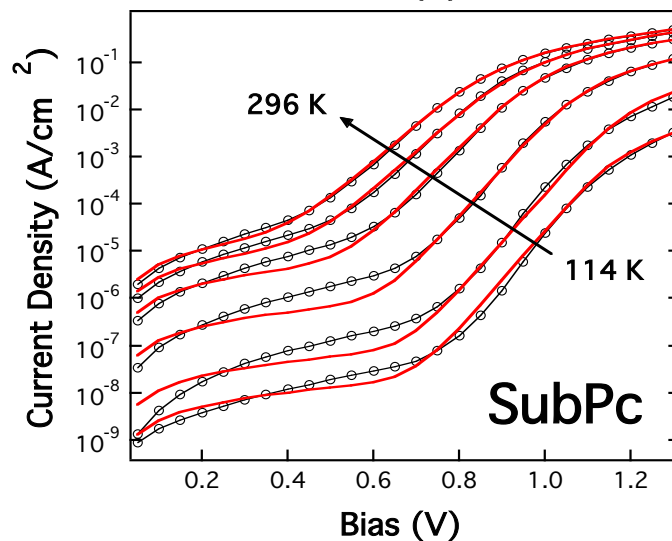
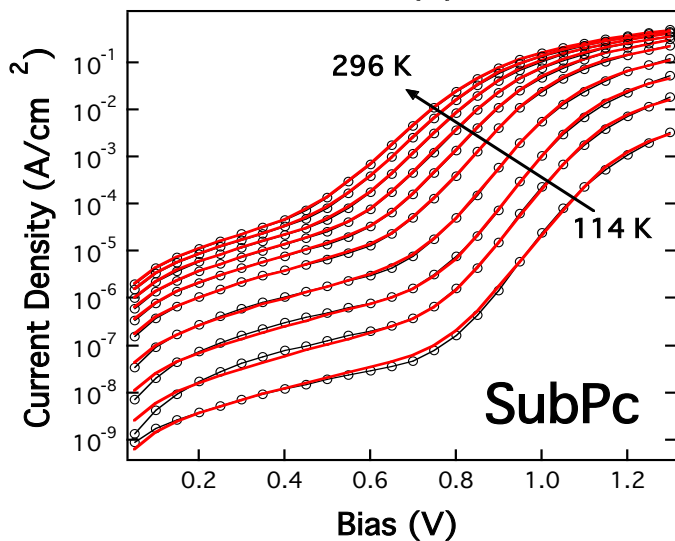
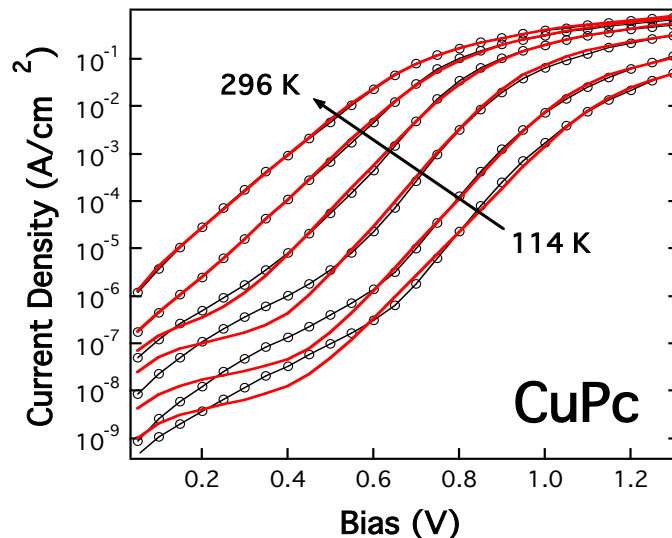
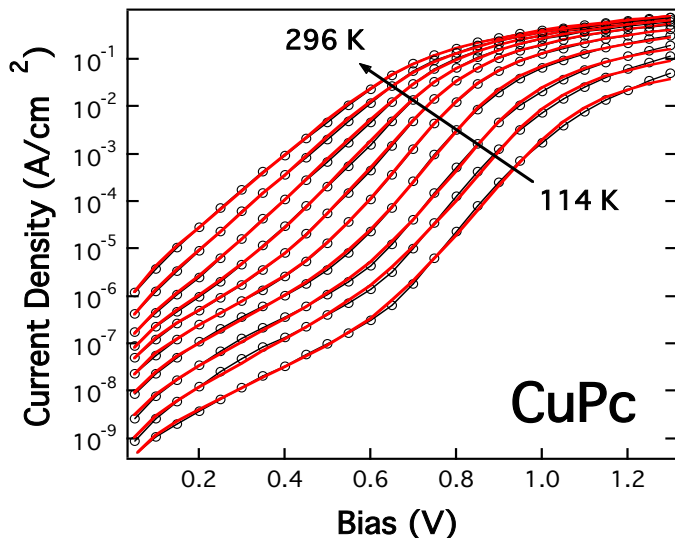
$$J = J_{sD} \left[\exp\left(\frac{q(V_a - JR_s)}{n_D k_b T}\right) - \frac{k_{PPd}}{k_{PPd,eq}} \right] + J_{sA} \left[\exp\left(\frac{q(V_a - JR_s)}{n_A k_b T}\right) - \frac{k_{PPd}}{k_{PPd,eq}} \right] - q\eta_{PPd} J_X$$



J-V Fits to Diode Eq. with Traps

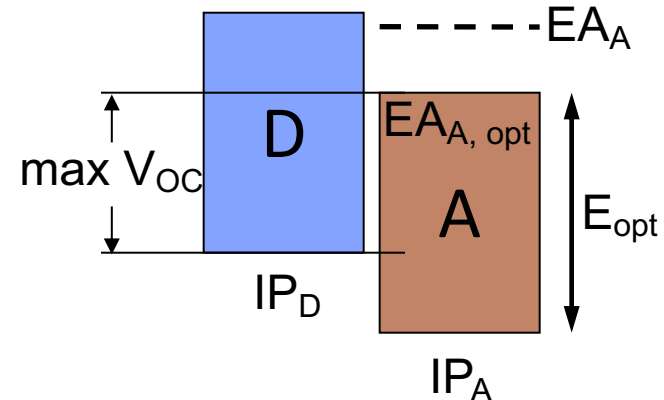
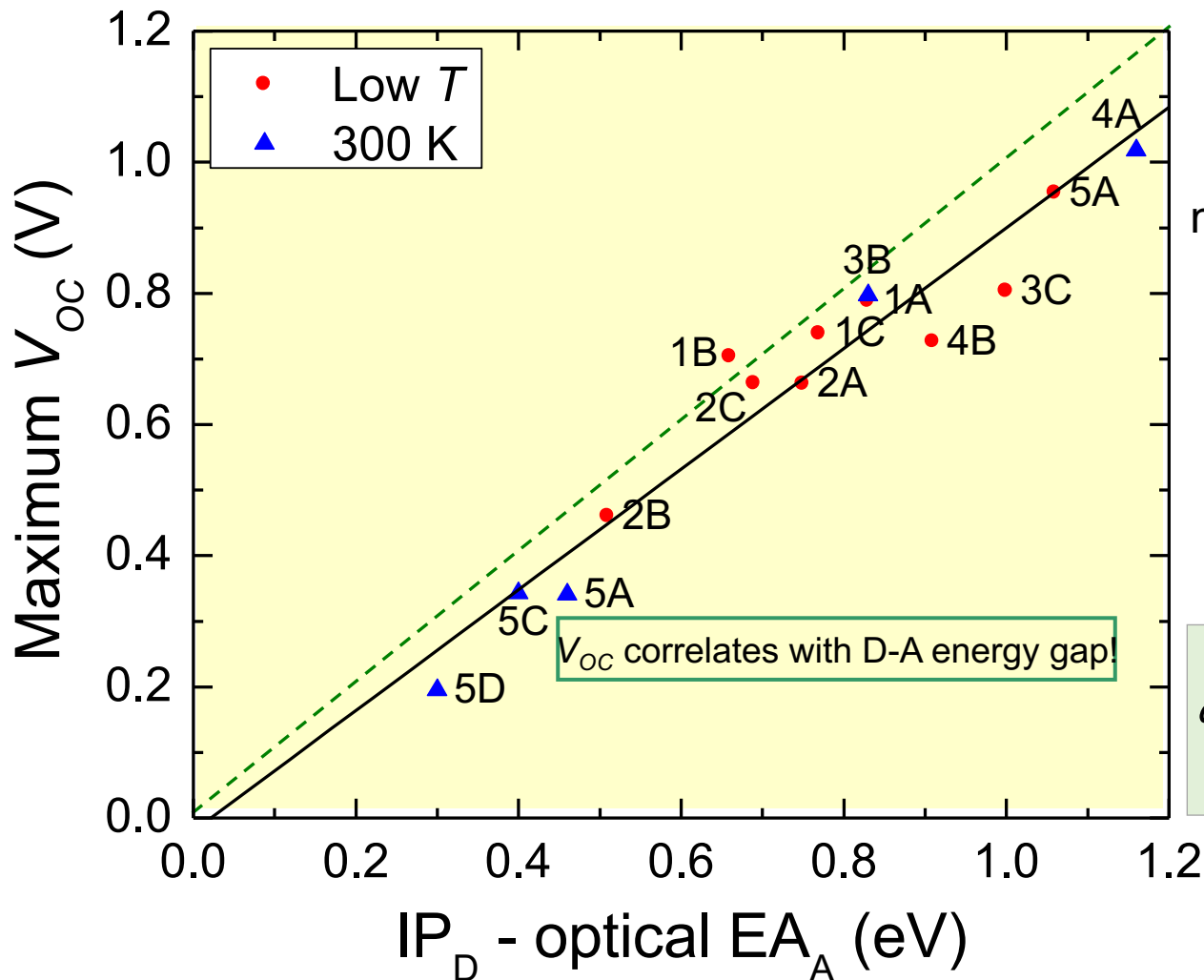
Org. HJ with Traps

Shockley Eq.



Acceptor
C₆₀

Dependence of V_{OC} on HJ Energies for Many Different D-A Combinations



A single rule fits all materials

$$qV_{OC}^{\max} = \underbrace{IP_D - EA_A}_{E_{HL}} - \underbrace{\frac{q^2}{4\pi\epsilon_0\epsilon_r r_{DA}}}_{-E_B}$$

PP Binding Energy

Stephen R. Forrest

What the theory tells us

Open Circuit Voltage

$$qV_{oc} = (\Delta E_{HL} - E_B) - k_b T \ln \left[\frac{k_{PPr} N_{HOMO} N_{LUMO}}{\zeta_{max} J_X / a_0} \right]$$

- At maximum sustainable power $J_x \sim a_0 N_{HOMO} k_{PPr}$
- More excitons cannot be supported.

Also: $\zeta_{max} \sim N_{HOMO} \sim N_{LUMO}$

Thus: $qV_{oc} = \Delta E_{HL} - E_B \rightarrow$ as observed!
(E_B =polaron energy)

- Slope under reverse bias due to PP recombination – eliminates R_p

Electronic Transport in Organics

-What we learned

- Origins of electronic band structure
- Concept of polarons (large and small)
- Charge transfer
- Conductivity, effective mass and mobility
- Doping
- Effects of trapped charge
- Schottky Barrier Diodes
- Organic Heterojunctions

